Comment on "Superwide-band negative refraction of a symmetrical E-shaped metamaterial with two electromagnetic resonances"

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The metamaterial presented by Yan *et al.* [Phys. Rev. E **77**, 056604 (2008)] is claimed to exhibit a superwide band of negative refraction. However, a retrieval procedure of the refractive index including the phase advance of a propagating plane wave shows that the material provides two distinct narrow bands with a negative refractive index instead of a single superwide band and thus the central claim of the paper is incorrect.

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Metamaterials with a negative refractive index play an important role for the realization of superlenses with subwavelength spatial resolution [1]. In order to achieve a negative index of refraction in a metamaterial with low absorption loss, the real parts of the permittivity and permeability of the effective medium are required to be simultaneously negative in the corresponding frequency band. Thereby, a negative permeability is usually implemented by use of subwavelength magnetic resonators which inherently provide a negative permeability only in a frequency band of limited width. However, when pulsed radiation is used where the spectral bandwidth of the pulse is significantly higher than for continuous-wave radiation, it is desirable to create a material with a negative refractive index over a superwide frequency band such that the complete pulse experiences a negative refractive index in the material.

In the paper of Yan *et al.* [2], a double E-shaped metamaterial is proposed for which the authors state that the effective index of refraction is negative in a very wide band, ranging from 47.8 GHz to at least 10 GHz, where the considered frequency range is cut-off. The statement is based on a parameter retrieval calculated from the simulated scattering (*S*) parameters of the structure.

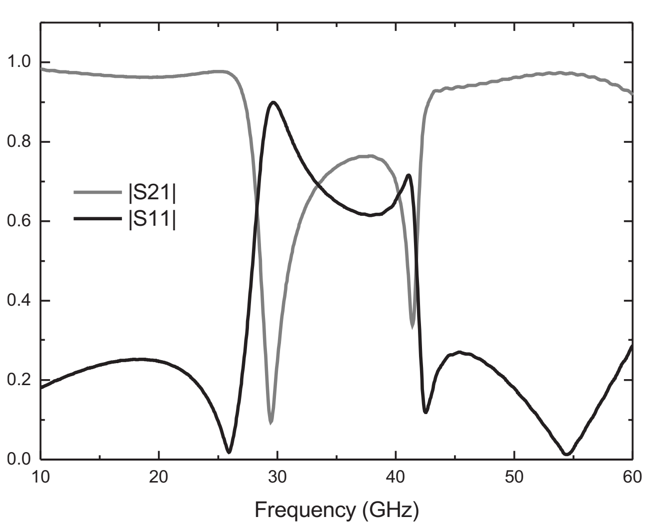

FIG. 1. Simulated *S* parameters of the metamaterial structure.

In order to reproduce this superwide left-handed behavior, we also simulated the *S* parameters of the structure with the same dimensions given in the paper. We considered one unit cell in the direction of propagation and periodic boundary conditions in the *x* and *y* directions, where the orientation of the coordinate system is the same as in Ref. [2]. The resulting *S* parameters shown in Fig. 1 are in good agreement with those presented in Ref. [2], confirming that our model adequately matches that of Yan *et al.*

To verify the negative index of refraction, we then applied a retrieval procedure [3] to calculate the effective refractive index from the simulated *S* parameters. The effective material parameters, which are intrinsic properties of the material, are independent of the thickness of the sample. In the retrieval algorithm, we used the *S* parameters of a structure with a thickness of four unit cells since the increasing of the sample thickness reduces the influence of surface effects and thus increases the reliability of the resulting effective parameters.

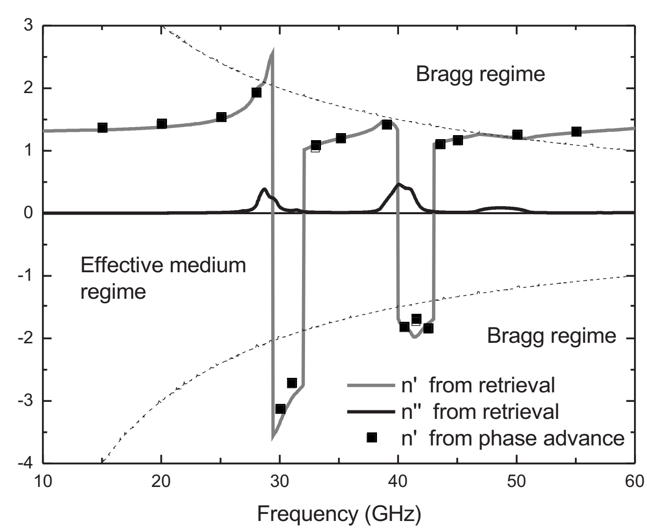

FIG. 2. Retrieved values for the refractive index, where $(\cdot)'$ and $(\cdot)''$ denote the real and imaginary parts, respectively. At the black squares, n' was calculated from the phase advance of a propagating plane wave. Outside the region framed by the two hyperbolas, the half wavelength inside the medium is smaller than the lattice constant and effective values for the refractive index are not expected to be reliable.

The correct branches of the complex logarithm function which comes up in the retrieval algorithm are chosen by simulating the phase progression of a plane wave while propagating through the metamaterial slab. For the correct branch, the resulting phase delay $\Delta\phi$ must agree with the value obtained for the real part of the retrieved refractive index n' according to

$$\Delta\phi = n' k_0 d,$$

where k_0 is the vacuum wave vector and d is the thickness of the slab. In our model, the phase of the electric field was evaluated along a path in the z direction. The path is centered in the y direction within a unit cell and is set a quarter lattice constant away from the metal parts in the x direction in order to avoid near-field effects.

The results presented in Fig. 2 reveal that the curve for the refractive index obtained by the retrieval procedure is in good agreement with the values calculated from the phase advance. This confirms that the calculated parameters are reliable. However, in contrast to Ref. [2], the real part of the refractive index is only negative within two narrow bands in the vicinity of the resonances of the structure. Moreover, in these regions, the half wavelength inside the medium is smaller than the lattice constant. This means that the structure actually operates in the Bragg regime, where the propagation is mainly determined by scattering and interference effects and the definition of an effective index of refraction is not justified.

We conclude that the retrieval procedure carried out by Yan *et al.* is erroneous and that the claim of a superwide left-handed band is not correct. We suppose that the branches of the $\cos^{-1}$ function in Eq. (1) of Ref. [2] are mistakenly chosen.

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